BME 665/565 Exam 1

Biophysical Models of Single Neurons

Due: February 20th, 2007

Do 4 of 5 problems. You must do all parts of the problems you choose.

1. Hodgkin-Huxley equations:

(A) In a single-compartment soma, use the Hodgkin-Huxley equations (Eq. 1) *without the potassium current* to determine the **two** equilibria for V. What is I_{Na} and (m,h) for each equilibrium state? ***Note:*** an analytic solution to this problem is difficult to derive. Using MATLAB to obtain a graphical solution is probably easiest, but you will need to make estimates of the conductances and reversal potentials to do so.

2. Post-inhibitory rebound:

(A) Show that post-inhibitory rebound (PIR) can arise from the Hodgkin-Huxley equations in a single compartment model without additional membrane currents:

$$C\frac{dV}{dt} = I_{stim} - g_L(V - V_L) - \bar{g}_{Na}m^3h(V - V_{Na}) - \bar{g}_K n^4(V - V_K)$$
$$\frac{dm}{dt} = \frac{1}{\tau_m}(m_\infty(V) - m), \quad \frac{dh}{dt} = \frac{1}{\tau_h}(h_\infty(V) - h), \quad \frac{dn}{dt} = \frac{1}{\tau_n}(n_\infty(V) - n), \qquad (1)$$

a) First argue using the voltage dependency and time course of *m, h,* and *n*

b) Simulate the effect using MATLAB (use parameter values from the table at the end).

(B) Let the leak current be caused exclusively by potassium ions. Could PIR be caused by a $GABA_A$ synapse? Use an α-function synapse,

$$g_\alpha(t) = \bar{g}_\alpha t e^{-t/\tau_\alpha} \qquad (2)$$

where the synaptic current is caused by chlorine ions. Vary the resting potential using V_L and the synaptic time constant, τ_α to determine if a $GABA_A$ synapse can cause PIR.

3. Nernst potentials:

(A) Calculate the Nernst potential for typical concentrations of calcium ions:

$$E_{Ca} = 12.5 \cdot \ln\frac{[Ca^{2+}]out}{[Ca^{2+}]in} \qquad (3)$$

(B) Compare the value of the Nerst equation (for V =-70 mV) to the Goldman-Hodgkin-Katz Current Equation for Ca^{2+}. Let the permeability of calcium, $P_{Ca}=10^{-4}$ cm/s.

(C) Calculate the change in E_{Ca} caused by an NMDA receptor opening during a spike that follows the presynaptic spike by 20 ms.

4. Cable properties:

(A) Show that, for a cable of finite length

$$\frac{\partial V_e}{\partial x} = -\frac{r_e}{r_e + r_i}\frac{\partial V}{\partial x} \quad (4)$$

where r_e and r_i are external and axoplasmic resistance/length, V_e is extracellular potential, and V is transmembrane potential. You will have to argue that $i_i = -i_e$ to derive Eqn 4.

5. Compartmental models:

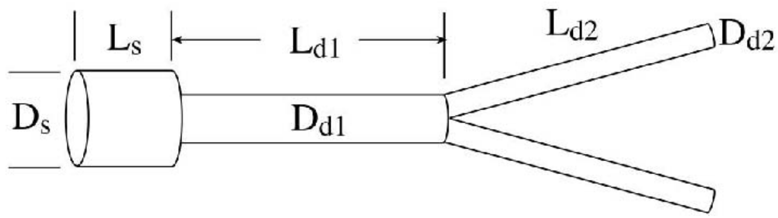

Figure 1: Morphology of neuron for problem 5.

Assume that a neuron has the morphology depicted in Fig. 1. All dendrites and the soma have Hodgkin-Huxley membrane currents; the soma has the default values listed in the table, and the dendrites have: $\bar{g}_{Na} = 0.05$ S/cm^2, $\bar{g}_K = 0.018$ S/cm^2. Let $D_s = 20\mu m$, $D_{d1} = 6\mu m$, $D_{d2} = 3\mu m$, $L_s = 20\mu m$, $L_{d1} = 60\mu m$, $L_{d2} = 100\mu m$.

a) Determine the threshold for back-propagating spikes.
b) What is the attenuation of a sub-threshold current pulse injected in the distal end of the dendrites.
c) If the distal dendrites have a diameter of $D_{d2} = 1\mu m$, what value of Dd1 would insure a spike generated in the dendrites would not propagate to the soma? What is the attenuation of a supra-threshold input at the distal end of the dendrite?

Table 1. Parameters for Hodgkin-Huxley simulations

Symbol	Definition	Values
V C I_{stim}	Membrane Potential Membrane capacitance Injected (stimulator) current	
g_L V_L	Leak (passive) conductance Leak (passive) reversal potential	g_L=0.0003 V_{pass}= -70, or V_{hh}= -54.3
$\bar{g}_{Na}$ V_{Na} m h	Sodium conductance Sodium reversal potential Sodium activation variable Sodium inactivation variable	$\bar{g}_{Na} = 0.12$ $V_{Na} = 50$ $m \to 1$ as $V \to 0$ $h \to 0$ as $V \to 0$
$\bar{g}_K$ V_K n	Potassium (delayed rectifier) conductance Potassium reversal potential Potassium activation variable	$\bar{g}_K = 0.036$ $V_K = -77$ $n \to 1$ as $V \to 0$