Assignment 4: Simple and noisy model neurons
Due: Thursday, March 1, 2007

Purpose:
1) To consolidate your understanding of the simplified model neurons discussed in class.
2) To investigate how noise affects simplified model neurons.

Deliverables:
Solutions to the questions below and appropriate graphical results of simulations performed with MatLab (or your favorite computational tool), together with an explanation of how you arrived at your results and an interpretation of your results. You are encouraged to work together to obtain the results, but you must submit your own writeup, discussion, and interpretation of those results.

1. *Integrate-and-fire neurons (analysis).* Write down the analytic solution of integrate-and-fire voltage equation,

$$\tau_m \frac{dV(t)}{dt} = (E_{leak} - V(t)) + R_m I(t), \tag{1}$$

when the injected current $I(t)$ is an arbitrary function of time. The solution will involve integrals that cannot be performed unless $I(t)$ is specified. Choose a nontrivial function for $I(t)$ that allows you to integrate and solve for $V(t)$.

2. *Integrate-and-fire neurons (simulation).* Add a voltage threshold, V_{thr}, to the model of Problem 1. Whenever the voltage crosses the threshold, set the voltage on that time step to $V_{spk} = +70$ mV to represent the generation of an action potential, and then reset the voltage to 0 mV on the next time step to represent the repolarization phase of the action potential.

(A) Using the following parameters: $R_m = 10$ MΩ, $\tau_m = 10$ msec, $V_{thr} = 5$ mV, $V_{spk} = 70$ mV, and a time step of 1 ms, simulate the response of the integrate-and-fire model. Generate a representative plot of the membrane voltage as a function of time for a step current injection with $I(t) = 1$ nA for $10 \leq t < 60$ ms, and $I(t) = 0$ otherwise.

(B) Examine the behavior of this model as you change the magnitude of the injection current. Compute a firing rate (spikes per second) by dividing the number of spikes by the duration of the stimulus. Generate a plot of "firing frequency vs. injected current" for currents from 0 to 10 nA. You'll need to determine a set of current injection values to use (make the spacing fine enough to map out the threshold transition). For each injection level, have your program count the number of spikes in a 1-second window to determine the firing rate (spikes/s). What is the minimum current that results in spike generation? Why does the firing rate saturate for large injection currents?

3. *Spike-response models (analysis).* Using the voltage equation for a spike-response model,

$$V(t) = \eta(t - t_{post}) + \sum_{pre} \epsilon(t - t_{pre}), \tag{2}$$

where t_{pre} is the time of presynaptic spikes, t_{post} is the time of postsynaptic spikes.

(A) The presynaptic spike-response kernel, $\epsilon(t)$ is an exponential decay function, $\epsilon(t) = (\epsilon_0/\tau_m)\exp[-t/\tau_m]$ for $t \geq 0$ and $\epsilon(t) = 0$ for $t < 0$. If the presynaptic spikes arrive at a constant rate, r, what is the minimum rate that generates a postsynaptic spike for a spike threshold of V_{thr}.

(B) The postsynaptic spike-response kernel, $\eta(t)$ is an exponential decay function, $\eta(t) = (-\eta_0/\tau_{recov})\exp[-t/\tau_{recov}]$ for $t \geq 0$ and $\eta(t) = 0$ for $t < 0$. If the presynaptic spikes arrive at a constant rate, r, what is the maximum rate that can be attained for a spike threshold of V_{thr}. Assume that the presynaptic neuron has a refractory period, T.

4. Generate a Poisson spike train with a time-dependent firing rate $r(t) = 100(1+\cos(2\pi t/300ms))$ Hz. Approximate the firing rate from this spike train using a variable r_{approx} that satisfies

$$\tau_{approx}\frac{d\,r_{approx}}{dt} = -r_{approx} \tag{3}$$

and $r_{approx} \rightarrow r_{approx} + 1/\tau_{approx}$ every time a spike occurs (this is like a postsynaptic potential). Make plots of the true rate, the spike sequence generated, and the approximated rate. Experiment with a few different values of τ_{approx} in the range of 1 to 100 ms. Determine the best value of τ_{approx} by computing the average squared error of the estimate, $\int dt(r(t) - r_{approx}(t))^2$, for different values of τ_{approx} , and finding the value of τ_{approx} that minimizes this error.